MINDLAB

IBDP Physics Revision Notes (First Assessment 2025)

Topic C.4: Standing Waves and Resonance

Syllabus Reference

Theme C: Wave Behaviour – Topic C.4 Standing Waves and Resonance
Teaching Hours: 4 hours (SL and HL – same content)
Note: There is no additional Higher Level content in C.4.

Guiding Questions

- What distinguishes standing waves from travelling waves?
- How does the form of standing waves depend on the boundary conditions?
- How can the application of force result in resonance within a system?

Learning Objectives

Students should understand:

- The nature and formation of standing waves in terms of superposition of two identical waves travelling in opposite directions
- Nodes and antinodes, relative amplitude and phase difference of points along a standing wave
- Standing wave patterns in strings and pipes
- The nature of resonance including natural frequency and amplitude of oscillation based on driving frequency
- The effect of damping on the maximum amplitude and resonant frequency of oscillation
- The effects of light, critical and heavy damping on the system

1 Formation of Standing Waves

Standing Wave

A **standing wave** (also called a stationary wave) is a wave pattern that results from the **superposition of two identical waves** travelling in **opposite directions**. The pattern appears to oscillate in place rather than propagate through space.
Important: A standing wave is a **phenomenon**, not a type of wave. Any wave type (transverse, longitudinal, or electromagnetic) can form standing waves.

Formation Mechanism

Standing waves form when:

1. A wave travels along a medium and reaches a boundary
2. The wave reflects at the boundary
3. The incident wave and reflected wave have the **same frequency and wavelength**
4. The waves travel in **opposite directions**
5. **Continuous superposition** creates a stationary pattern of nodes and antinodes

Standing Wave vs Travelling Wave

Travelling (Progressive) Wave	**Standing Wave**
Energy propagates through space	Energy is confined to the system
All points oscillate with the same amplitude	Different points oscillate with different amplitudes
Wave pattern moves through the medium	Wave pattern appears stationary
All points pass through equilibrium at different times	Points between nodes pass through equilibrium simultaneously
No permanently stationary points	Nodes are permanently at rest

Critical insight: The "velocity" $v = f\lambda$ in standing wave equations refers to the component travelling waves, not the movement of the standing wave pattern (which does not move).

2 Nodes and Antinodes

Nodes and Antinodes

Node (N): A point of **zero amplitude** where destructive interference always occurs.

- Particles at nodes do not oscillate
- Fixed/closed boundaries must be nodes

Antinode (A): A point of **maximum amplitude** where constructive interference occurs.

- Particles oscillate with maximum displacement
- Free/open boundaries are antinodes

Spatial Relationships in Standing Waves

These relationships are **NOT** in the Data Booklet – you must know them:

$$\text{Distance between consecutive nodes} = \frac{\lambda}{2}$$

$$\text{Distance between consecutive antinodes} = \frac{\lambda}{2}$$

$$\text{Distance from node to adjacent antinode} = \frac{\lambda}{4}$$

Standing Wave Pattern

A standing wave on a string at different instants:
At maximum displacement (solid line):

- Draw a sinusoidal curve with peaks and troughs
- Mark positions of zero displacement as N (nodes)
- Mark positions of maximum displacement as A (antinodes)

At equilibrium (dashed line):

- All points on the equilibrium line (zero displacement)
- Nodes remain at the same positions

At opposite maximum (solid line, mirrored):

- Previous peaks become troughs and vice versa
- Nodes remain stationary

Key measurements to show:

- $\lambda/2$ between consecutive nodes
- $\lambda/2$ between consecutive antinodes
- $\lambda/4$ from node to adjacent antinode

3 Phase Relationships in Standing Waves

Phase Difference

Between adjacent nodes:

- All points oscillate **in phase** ($\Delta\phi = 0$ radians)
- They all reach maximum, equilibrium, and minimum together

On opposite sides of a node:

- Points oscillate **out of phase** ($\Delta\phi = \pi$ radians or 180)
- When one region moves up, the other moves down

Note: All points on a standing wave oscillate at the **same frequency**, regardless of their amplitude.

4 Standing Waves in Strings

Boundary Conditions for Strings

Fixed end: Must be a **node**

- Particles cannot move at the fixed point
- Reflection occurs with phase inversion (π phase change)

Free end: Must be an **antinode**

- Maximum displacement is possible
- Reflection occurs without phase inversion

4.1 String Fixed at Both Ends

Both Ends Fixed – Harmonic Relationships

For a string of length L fixed at both ends:
Wavelength:

$$\boxed{\lambda_n = \frac{2L}{n}}$$

Frequency:

$$\boxed{f_n = \frac{nv}{2L} = nf_1}$$

where:

- $n = 1, 2, 3, 4, 5, \ldots$ (all positive integers – **all harmonics present**)
- v = wave speed on the string (m/s)
- L = length of the string (m)
- $f_1 = \frac{v}{2L}$ is the first harmonic frequency

Harmonics for String Fixed at Both Ends

1st Harmonic ($n = 1$):

- Nodes at both ends only
- 1 antinode in the middle
- $L = \lambda_1/2$, so $\lambda_1 = 2L$
- $f_1 = v/2L$

2nd Harmonic ($n = 2$):

- Nodes at both ends plus 1 internal node (center)
- 2 antinodes
- $L = \lambda_2$, so $\lambda_2 = L$
- $f_2 = v/L = 2f_1$

3rd Harmonic ($n = 3$):

- Nodes at both ends plus 2 internal nodes
- 3 antinodes
- $L = 3\lambda_3/2$, so $\lambda_3 = 2L/3$
- $f_3 = 3v/2L = 3f_1$

Pattern: The nth harmonic has n antinodes and $(n + 1)$ nodes.

Drawing and Deriving Harmonic Relationships

Step 1: Identify boundary conditions (fixed or free at each end)
Step 2: Draw boundaries (node at fixed, antinode at free)
Step 3: Draw the harmonic pattern

- 1st harmonic: Simplest pattern satisfying boundary conditions
- Higher harmonics: Add one more node each time

Step 4: Count wavelengths

- How many half-wavelengths ($\lambda/2$) fit in length L?

Step 5: Write the relationship: $L = n \times \frac{\lambda}{2}$ or $L = n \times \frac{\lambda}{4}$
Step 6: Solve for λ
Step 7: Use $v = f\lambda$ to find frequency
Key principle: Draw first, derive second, calculate last!

5 Standing Waves in Pipes (Air Columns)

Boundary Conditions for Pipes

The IB uses **displacement nodes and antinodes** only (not pressure).
Closed end: Must be a displacement **node**

- Air particles cannot move through the closed boundary

Open end: Must be a displacement **antinode**

- Air particles are free to oscillate at the opening

Note: End corrections for open pipes are **not required** for the IB exam.

5.1 Pipe Open at Both Ends

Open Pipe – Harmonic Relationships

For a pipe of length L open at both ends:
Wavelength:

$$\boxed{\lambda_n = \frac{2L}{n}}$$

Frequency:

$$\boxed{f_n = \frac{nv}{2L} = nf_1}$$

where $n = 1, 2, 3, 4, 5, \ldots$ (**all harmonics present**)
This is identical to a string fixed at both ends (both have the same type of boundary at each end).

5.2 Pipe Closed at One End

Closed Pipe – Harmonic Relationships

For a pipe of length L closed at one end and open at the other:
Wavelength:

$$\boxed{\lambda_n = \frac{4L}{n}}$$

Frequency:

$$\boxed{f_n = \frac{nv}{4L} = nf_1}$$

where $n = 1, 3, 5, 7, 9, \ldots$ (**only odd harmonics present**)
Critical: There is no 2nd, 4th, 6th harmonic, etc. The sequence jumps: 1st → 3rd → 5th → 7th...

Harmonics for Pipe Closed at One End

1st Harmonic ($n = 1$):

- Node at closed end, antinode at open end
- 1 antinode total
- $L = \lambda_1/4$, so $\lambda_1 = 4L$
- $f_1 = v/4L$

3rd Harmonic ($n = 3$):

- Node at closed end, antinode at open end
- 2 antinodes total
- $L = 3\lambda_3/4$, so $\lambda_3 = 4L/3$
- $f_3 = 3v/4L = 3f_1$

5th Harmonic ($n = 5$):

- Node at closed end, antinode at open end
- 3 antinodes total
- $L = 5\lambda_5/4$, so $\lambda_5 = 4L/5$
- $f_5 = 5v/4L = 5f_1$

Why no even harmonics?
An even harmonic pattern would require a node at the open end or an antinode at the closed end – both violate the boundary conditions!

6 Comparison of Open and Closed Pipes

Property	**Open Pipe (Both Ends Open)**	**Closed Pipe (One End Closed)**
Boundary conditions	Antinode at both ends	Node at closed end, antinode at open end
Harmonics present	All: $n = 1, 2, 3, 4, \ldots$	Odd only: $n = 1, 3, 5, 7, \ldots$
Wavelength formula	$\lambda_n = \frac{2L}{n}$	$\lambda_n = \frac{4L}{n}$
Frequency formula	$f_n = \frac{nv}{2L}$	$f_n = \frac{nv}{4L}$
First harmonic wavelength	$\lambda_1 = 2L$	$\lambda_1 = 4L$
First harmonic frequency	$f_1 = \frac{v}{2L}$	$f_1 = \frac{v}{4L}$
Sound quality	Rich, bright (all harmonics)	Hollow, mellow (missing even harmonics)
Examples	Flute, recorder, open organ pipes	Clarinet, closed organ pipes

Terminology Note

The IB Physics Guide 2025 states:

> "The lowest frequency mode of a standing wave will be referred to as the **first harmonic**. The terms fundamental and overtone are not to be used in this course."

Always use "first harmonic," "second harmonic," etc. Never use "fundamental" or "overtone."

7 Resonance

Natural Frequency and Resonance

Natural frequency (f_0): The frequency at which a system naturally oscillates when disturbed and left alone. It is determined by the physical properties of the system.
Driving frequency (f_D): The frequency of an external periodic force applied to the system. A driven system oscillates at the driving frequency, not its natural frequency.
Resonance: A phenomenon where the amplitude of oscillation becomes maximum when the driving frequency equals the natural frequency:

$$\boxed{f_D = f_0 \text{ (resonance condition)}}$$

At resonance, energy transfer from the driving force to the oscillating system is most efficient.

Resonance Characteristics

- Small periodic forces can produce large amplitude oscillations
- Energy input is maximally efficient at resonance
- The system absorbs energy at its maximum rate
- Each harmonic of a standing wave represents a natural frequency of the system

Examples of resonance:

- Pushing a child on a swing at the right frequency
- Tuning fork causing resonance in an air column
- Breaking a wine glass with sound at its natural frequency
- Musical instruments producing sound

Resonance Amplitude-Frequency Graph

Axes:

- Horizontal axis: Driving frequency (f_D)
- Vertical axis: Amplitude of oscillation

Shape:

- A peaked curve centered on f_0 (natural frequency)
- Maximum amplitude occurs at $f_D = f_0$
- Amplitude decreases as f_D moves away from f_0

Effect of damping (show multiple curves):

- **Light damping:** Sharp, tall peak at f_0
- **Medium damping:** Lower peak, shifted slightly left, broader curve
- **Heavy damping:** Very low peak, shifted more left, very broad curve

Key features to label: f_0, maximum amplitude, width of peak

Useful and Destructive Effects of Resonance

Useful effects:

- Musical instruments produce and amplify sound
- Tuning radio/TV receivers to specific frequencies
- Microwave ovens excite water molecules at resonance
- MRI machines use nuclear magnetic resonance for imaging

Destructive effects:

- Tacoma Narrows Bridge collapse (1940) – wind at natural frequency
- Building damage during earthquakes when seismic frequency matches natural frequency
- Mechanical failure in machinery due to vibration at resonance
- Structural damage from marching soldiers on bridges

Engineers must design structures to avoid resonance with expected driving forces.

8 Damping

Damping

Damping refers to resistive forces (friction, air resistance, internal friction) that remove energy from an oscillating system. Damping causes:

- Mechanical energy to be converted to thermal energy
- Amplitude to decrease over time
- Oscillations to eventually cease

8.1 Types of Damping

Type	Description	Behavior	Example
Light damping (underdamped)	Small resistive forces	Gradual amplitude decrease; many oscillations before stopping	Pendulum in air; tuning fork
Critical damping	Minimum damping to prevent oscillation	Fastest return to equilibrium **without overshooting**; no oscillation	Ideal car shock absorbers
Heavy damping (overdamped)	Large resistive forces	Slow return to equilibrium; no oscillation; takes longer than critical	Door closer mechanism; motion in viscous fluid

Common Misconception About Critical Damping

Critical damping is NOT maximum damping!
The order of damping strength is:

$$\text{Light (weak)} \rightarrow \text{Critical (medium)} \rightarrow \text{Heavy (strong)}$$

Critical damping is the **optimal** amount – just enough to prevent oscillation while returning to equilibrium as quickly as possible. Heavy damping returns more slowly.

Displacement-Time Graphs for Different Damping Types

All three curves start at the same initial displacement:
Light damping (blue curve):

- Oscillates back and forth across equilibrium
- Amplitude gradually decreases (exponential envelope)
- Many cycles before stopping

Critical damping (green curve):

- Returns directly to equilibrium without crossing it
- Fastest curve to reach equilibrium
- No oscillation

Heavy damping (red curve):

- Returns to equilibrium without crossing it
- Slower than critical damping
- No oscillation

Label: equilibrium position (x = 0), initial displacement, time

8.2 Effect of Damping on Resonance

Damping Effects on Resonance Curves

As damping increases:

1. **Maximum amplitude decreases** – the peak height reduces
2. **Resonant frequency shifts slightly lower** – peak moves left
3. **Resonance peak becomes broader** – less sharp selectivity

Note: Only a **qualitative analysis** is required. You do not need to calculate damping coefficients.

9 Worked Examples

String Harmonic Calculation

A guitar string of length 0.64 m is fixed at both ends. The speed of waves on the string is 256 m/s.
(a) Calculate the wavelength of the first harmonic.
(b) Calculate the frequency of the first harmonic.
(c) Calculate the frequency of the third harmonic.
Solution:
Given: $L = 0.64$ m, $v = 256$ m/s, both ends fixed
(a) First harmonic wavelength:

Draw the first harmonic: nodes at both ends, one antinode in the middle.

Half a wavelength fits in the length: $L = \frac{\lambda_1}{2}$

$$\lambda_1 = 2L = 2 \times 0.64 = \boxed{1.28 \text{ m}}$$

(b) First harmonic frequency:

Using $v = f\lambda$:

$$f_1 = \frac{v}{\lambda_1} = \frac{256}{1.28} = \boxed{200 \text{ Hz}}$$

Alternatively: $f_1 = \frac{v}{2L} = \frac{256}{2 \times 0.64} = 200$ Hz

(c) Third harmonic frequency:

For a string fixed at both ends: $f_n = nf_1$

$$f_3 = 3f_1 = 3 \times 200 = \boxed{600 \text{ Hz}}$$

Closed Pipe Calculation

An organ pipe is 5.16 m long, closed at one end and open at the other. The speed of sound in air is 344 m/s.

(a) Calculate the frequency of the first harmonic.

(b) Calculate the frequency of the third harmonic.

(c) Explain why there is no second harmonic.

Solution:

Given: $L = 5.16$ m, $v = 344$ m/s, one end closed

(a) First harmonic frequency:

Draw the first harmonic: node at closed end, antinode at open end.

A quarter wavelength fits in the length: $L = \frac{\lambda_1}{4}$

$$\lambda_1 = 4L = 4 \times 5.16 = 20.64 \text{ m}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{344}{20.64} = \boxed{16.7 \text{ Hz}}$$

(b) Third harmonic frequency:

For a pipe closed at one end with odd harmonics: $f_n = nf_1$

$$f_3 = 3f_1 = 3 \times 16.7 = \boxed{50.1 \text{ Hz}}$$

Or using the formula: $f_3 = \frac{3v}{4L} = \frac{3 \times 344}{4 \times 5.16} = 50.0$ Hz

(c) Explanation:

A second harmonic would require a pattern where $L = \frac{\lambda_2}{2}$. This would create a node at one end and another node at the other end. However, the boundary condition requires an antinode at the open end, not a node.

Therefore, the second harmonic pattern cannot satisfy the boundary conditions, and **only odd harmonics** (1st, 3rd, 5th, ...) can exist in a pipe closed at one end.

Resonance Tube Experiment

A student uses a tuning fork of frequency 512 Hz with a resonance tube partially submerged in water. The first loud resonance occurs when the air column length is 16.5 cm. The second loud resonance occurs at 50.5 cm.
(a) Calculate the wavelength of sound in air.
(b) Calculate the speed of sound in air.
Solution:
Given: $f = 512$ Hz, $L_1 = 0.165$ m (first resonance), $L_3 = 0.505$ m (second resonance)
(a) Wavelength:
The tube with water acts as a pipe closed at one end (water surface = closed, top = open).
First resonance = 1st harmonic: $L_1 = \frac{\lambda}{4}$
Second resonance = 3rd harmonic: $L_3 = \frac{3\lambda}{4}$
The difference: $L_3 - L_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{2\lambda}{4} = \frac{\lambda}{2}$

$$\frac{\lambda}{2} = L_3 - L_1 = 0.505 - 0.165 = 0.340 \text{ m}$$

$$\lambda = 2 \times 0.340 = \boxed{0.680 \text{ m}}$$

(b) Speed of sound:
Using $v = f\lambda$:

$$v = 512 \times 0.680 = \boxed{348 \text{ m/s}}$$

Note: Using the difference method ($L_3 - L_1$) helps eliminate systematic errors from end corrections.

Natural Frequency and Resonance

A mass-spring system has a natural frequency of 2.0 Hz. The system is driven by an external force.
(a) At what driving frequency will resonance occur?
(b) Describe what happens to the amplitude if the driving frequency is increased to 3.0 Hz.
(c) How would adding light damping affect the system at resonance?
Solution:
(a) Resonance occurs when driving frequency equals natural frequency:

$$f_D = f_0 = \boxed{2.0 \text{ Hz}}$$

(b) When $f_D = 3.0$ Hz, the driving frequency is greater than the natural frequency. The amplitude will be **significantly smaller** than at resonance because the system cannot respond efficiently to the driving force. Energy transfer is less effective when $f_D \neq f_0$.
(c) With light damping:

- The maximum amplitude at resonance will be **reduced** (but still relatively large)
- The resonant frequency will shift **slightly lower**
- The resonance peak will become **slightly broader**
- The system will reach a steady-state amplitude where energy input equals energy dissipated

10 Common Pitfalls and Examiner Tips

Frequent Errors to Avoid

1. **Distance between nodes $\neq$ wavelength**
   - Distance between consecutive nodes = $\lambda/2$, NOT λ
   - Always remember: wavelength is the distance for one complete cycle
2. **Expecting even harmonics in a closed pipe**
   - Only odd harmonics exist: $n = 1, 3, 5, 7, \ldots$
   - Cannot find f_2 or f_4 for a pipe closed at one end
3. **Using wrong formula for boundary type**
   - Both ends same type: $\lambda_n = 2L/n$
   - Different end types: $\lambda_n = 4L/n$
   - Always draw the pattern first!
4. **Confusing harmonic number with number of antinodes**
   - For closed pipes, 3rd harmonic has only 2 antinodes
   - Harmonic number = frequency ratio to first harmonic
5. **Thinking critical damping is maximum**
   - Critical is between light and heavy
   - Critical = fastest return without oscillation
6. **Using "fundamental" or "overtone"**
   - IB 2025 requires: "first harmonic," "second harmonic," etc.
   - These terms will not appear in correct exam answers

IB Examination Advice

- **Draw before calculating:** Always sketch the standing wave pattern before applying formulas. This helps identify the correct relationship between L and λ.
- **Show clear working:** Even for simple calculations, write out the formula, substitution, and answer with units.
- **Derive formulas from diagrams:** These formulas are NOT in the Data Booklet. Be prepared to derive $\lambda_n = 2L/n$ or $\lambda_n = 4L/n$ by counting wavelengths in your diagram.
- **Phase questions:** When asked about phase difference, state clearly whether points are "in phase" ($\Delta\phi = 0$) or "out of phase" ($\Delta\phi = \pi$ radians).
- **Resonance curves:** Know how to sketch and interpret amplitude vs. frequency graphs, including the effects of damping.
- **Damping descriptions:** Use qualitative language only – no damping coefficient calculations required.
- **Applications:** Be prepared to discuss both useful (musical instruments, MRI) and destructive (bridges, buildings) effects of resonance.
- **Units:** Frequency in Hz, wavelength in m, velocity in m/s. Convert cm to m before calculations.

11 Topic Links and Connections

Links to Other Topics

C.1 Simple Harmonic Motion:

- Natural frequency concept originates from SHM
- For a mass-spring system: $f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
- Damping in SHM leads to the same three regimes (light, critical, heavy)

C.2 Wave Model:

- Standing waves result from superposition principle
- Wave equation $v = f\lambda$ applies to the component travelling waves
- Phase relationships build on wave phase concepts

C.3 Wave Phenomena:

- Reflection at boundaries is essential for standing wave formation
- Phase change on reflection determines boundary conditions
- Interference (constructive/destructive) creates nodes and antinodes

B.2 Greenhouse Effect (NOS):

- Greenhouse gas molecules (CO_2, H_2O, CH_4) have natural vibration frequencies
- Infrared radiation at matching frequencies is absorbed through resonance
- Molecular resonance explains selective absorption of IR radiation

A.3 Work, Energy and Power:

- Damping converts mechanical energy to thermal energy
- At resonance, energy transfer is most efficient
- Conservation of energy applies to driven oscillators

Looking ahead – Topic E (Quantum Physics):

- Electrons in atoms form standing waves
- de Broglie wavelength: $\lambda = h/p$
- Allowed energy levels correspond to standing wave conditions

12 Diagram Practice

Use these spaces to practice drawing essential diagrams for this topic.

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13 Quick Reference Summary

Essential Equations (Not in Data Booklet)

Wave equation (in Data Booklet):

$$v = f\lambda$$

Spatial relationships:

$$\text{Node to node} = \frac{\lambda}{2}$$
$$\text{Antinode to antinode} = \frac{\lambda}{2}$$
$$\text{Node to antinode} = \frac{\lambda}{4}$$

Both ends same type (fixed-fixed or open-open):

$$\lambda_n = \frac{2L}{n}, \quad f_n = \frac{nv}{2L} = nf_1 \quad (n = 1, 2, 3, 4, \ldots)$$

One end different (closed-open):

$$\lambda_n = \frac{4L}{n}, \quad f_n = \frac{nv}{4L} = nf_1 \quad (n = 1, 3, 5, 7, \ldots)$$

Resonance condition:

$$f_D = f_0$$

14 Practice Questions

1. A string of length 1.2 m is fixed at both ends. The speed of waves on the string is 300 m/s.
   - (a) Calculate the frequency of the first harmonic.
   - (b) Calculate the wavelength of the fourth harmonic.
   - (c) What is the frequency ratio of the fourth harmonic to the second harmonic?
2. A pipe is 0.85 m long and closed at one end.
   - (a) Sketch the first and third harmonics for this pipe, clearly showing nodes and antinodes.
   - (b) Given the speed of sound is 340 m/s, calculate the frequencies of the first and third harmonics.
   - (c) Explain why the second harmonic cannot exist in this pipe.
3. Two points A and B are on a standing wave. Point A is at an antinode and point B is located $\lambda/4$ from A.
   - (a) What type of point is B (node or antinode)?
   - (b) What is the phase difference between points on either side of B?
4. A system has a natural frequency of 5.0 Hz.
   - (a) At what driving frequency will maximum amplitude occur?
   - (b) Describe and explain how the behavior changes if heavy damping is added.

5. Distinguish between light, critical, and heavy damping in terms of the motion of an oscillating system.

Answers:

1. (a) 125 Hz (b) 0.60 m (c) 2:1 (or $f_4 = 2f_2$)
2. (b) $f_1 = 100$ Hz, $f_3 = 300$ Hz (c) Even harmonics require wrong boundary conditions
3. (a) Node (b) π radians (or 180)
4. (a) 5.0 Hz (b) Maximum amplitude much smaller, resonance peak broader, slightly lower resonant frequency, no oscillation – slow return to equilibrium
5. Light: oscillates many times, gradual decay; Critical: fastest return without oscillation; Heavy: slow return, no oscillation

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End of Topic C.4: Standing Waves and Resonance